PC12

Chapter 6 Review

NAME:

6.1 Reciprocal, Quotient, and Pythagorean Identities, pages 188–196

1. Determine the non-permissible values of x, in radians, for each expression.

   a) $\dfrac{\sec x}{\sin x}$

   b) $\dfrac{\cos x}{\csc x}$

   c) $\dfrac{\sec x}{1 + \cos^2 x}$

2. Simplify each expression to one of the three primary trigonometric functions: $\sin x$, $\cos x$, or $\tan x$.

   a) $\dfrac{\cos x \csc x}{\sec x \cot x}$

   b) $\dfrac{\cot x \tan x}{\csc x}$

3. Simplify. Then, rewrite each expression as one of the three reciprocal trigonometric functions: $\csc x$, $\sec x$, or $\cot x$.

   a) $\cot x \sec x$

   b) $\dfrac{\cos x}{(1 - \sin x)(1 + \sin x)}$

4. a) Verify that the equation $(\sec x + \tan x) \cos x - 1 = \sin x$ is true for $x = 30°$ and for $x = \frac{\pi}{3}$.

   b) What are the non-permissible values of the equation in part a) in the domain $0° \leq x < 360°$?

6.2 Sum, Difference, and Double-Angle Identities, pages 197–204

5. Write each expression as a single trigonometric ratio. Then, give an exact value for the expression.

   a) $\cos^2 15° - \sin^2 15°$

   b) $\sin 35° \cos 100° + \cos 35° \sin 100°$

   c) $1 - 2\sin^2 75°$

6. Determine the exact value of each trigonometric expression.

   a) $\sin\left(-\frac{\pi}{12}\right)$

   b) $\cos\frac{\pi}{12}$

   c) $\cos 105°$

   d) $\sin\frac{23\pi}{12}$

7. Angle θ is in quadrant II and $\sin\theta = \frac{7}{25}$. Determine an exact value for each of the following.

   a) $\sin 2\theta$

   b) $\cos 2\theta$

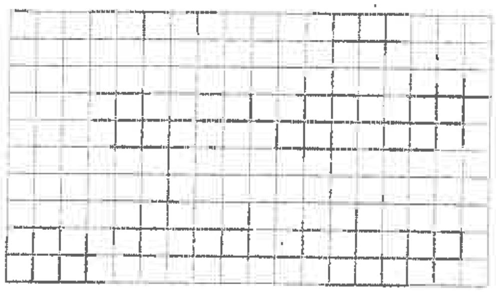
   c) $\tan 2\theta$

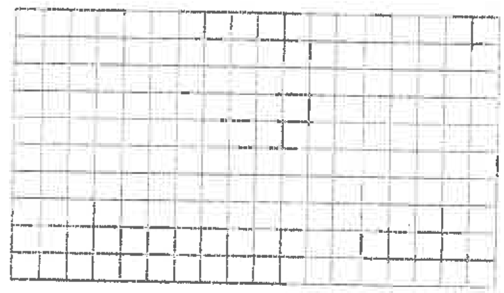
6.3 Proving Identities, pages 205–214

8. Prove $\sin(\pi - x) - \tan(\pi + x) = \dfrac{\sin x(\cos x - 1)}{\cos x}$.

9. Consider the equation $\sin^2 x + \tan^2 x + \cos^2 x = \sec^2 x$.

   a) Graph each side of the equation. Could the equation be an identity? Explain.





   b) Either prove that the equation is an identity or find a counterexample to show that it is not an identity.

10. Prove each identity.

   a) $\cos x \tan^2 x = \sin x \tan x$

   b) $\sin 2x = \tan x + \tan x \cos 2x$

6.4 Solving Trigonometric Equations Using Identities, pages 215–223

11. Rewrite each equation in terms of sine or cosine. Then, solve algebraically for $0 \leq x < 2\pi$.

a) $2\cos^2 x - \sin x = -1$

b) $\sin^2 x = 2\cos x - 2$

c) $\cos x + \cos 2x = 0$

12. Solve each equation algebraically over the domain $0 \leq x < 2\pi$.

a) $2\cos^2 x + 3\cos x + 1 = 0$

b) $\sin^2 x + 3\sin x + 2 = 0$

c) $\sin^2 x + 5\sin x + 6 = 0$

d) $\cos^2 x + 3\cos x + 2 = 0$

13. Solve the equation $2\cos^2 x = 1 - \sin x$ algebraically. Give the general solution expressed in radians.

13. The step that is wrong in the solution is the line $\cos x(\cos x + 1) = 0$. Brooke mistakenly thought that $\cos 2x$ meant $\cos^2 x$. The correct solution is:

$$\cos 2x + \cos x = 0$$
$$2\cos^2 x - 1 + \cos x = 0$$
$$2\cos^2 x + \cos x - 1 = 0$$
$$(2\cos x - 1)(\cos x + 1) = 0$$
$$\cos x = \frac{1}{2} \text{ or } \cos x = -1$$

For $\cos x = \frac{1}{2}$: $x = 60° + 360°n$ or $x = 300° + 360°n$, where $n \in I$

For $\cos x = -1$: $x = 180° + 360°n$, where $n \in I$

KEY

Chapter 6 Review, pages 224–227

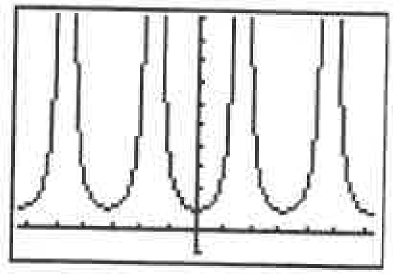
1. a) $x \neq \frac{\pi}{2}n$, where $n \in I$
   b) $x \neq \pi n$, where $n \in I$
   c) $x \neq \frac{\pi}{2} + \pi n$, where $n \in I$
2. a) $\cos x$ b) $\sin x$
3. a) $\csc x$ b) $\sec x$
4. a) When substituted, both values satisfy the equation.
   b) $x \neq 90°, 270°$
5. a) $\cos 30° = \frac{\sqrt{3}}{2}$
   b) $\sin 135° = \frac{\sqrt{2}}{2}$
   c) $\cos 150° = -\frac{\sqrt{3}}{2}$
6. a) $\frac{\sqrt{2} + \sqrt{6}}{4}$ b) $\frac{\sqrt{6} + \sqrt{2}}{4}$
   c) $\frac{\sqrt{2} - \sqrt{6}}{4}$ d) $\frac{-\sqrt{6} + \sqrt{2}}{4}$
7. a) $-\frac{336}{625}$ b) $\frac{527}{625}$
   c) $-\frac{336}{527}$
8. Left Side $= \sin(\pi - x) - \tan(\pi + x)$

$$= \sin \pi \cos x - \sin x \cos \pi - \frac{\tan \pi + \tan x}{1 - \tan \pi \tan x}$$
$$= \sin x - \tan x$$

Right Side $= \frac{\sin x(\cos x - 1)}{\cos x}$

$$= \frac{\sin x \cos x - \sin x}{\cos x}$$
$$= \frac{\sin x \cos x}{\cos x} - \frac{\sin x}{\cos x}$$
$$= \sin x - \tan x$$

Left Side = Right Side

9. a) 

Yes. The graphs appear to be equal.

b) Left Side $= \sin^2 x + \tan^2 x + \cos^2 x$
$$= 1 + \tan^2 x$$
$$= \sec^2 x$$
= Right Side

10. a) Left Side $= \cos x \tan^2 x$
$$= \cos x \frac{\sin^2 x}{\cos^2 x}$$
$$= \frac{\sin^2 x}{\cos x}$$
$$= \sin x \frac{\sin x}{\cos x}$$
$$= \sin x \tan x$$
= Right Side

b) Right Side $= \tan x + \tan x \cos 2x$
$$= \tan x(1 + \cos 2x)$$
$$= \tan x(1 + (2\cos^2 x - 1))$$
$$= \tan x(2\cos^2 x)$$
$$= 2\frac{\sin x}{\cos x}\cos^2 x$$
$$= 2\sin x \cos x$$
$$= \sin 2x$$
= Left Side

11. a) $\frac{\pi}{2}$ b) 0
    c) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$
12. a) $\frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ b) $\frac{3\pi}{2}$
    c) no solution d) π
13. The general solution is $x = \frac{7\pi}{6} + 2\pi n$, $x = \frac{11\pi}{6} + 2\pi n$, $x = \frac{\pi}{2} + 2\pi n$, where $n \in I$. These can be combined as $x = \frac{\pi}{2} + \frac{2\pi}{3}n$, where $n \in I$.