Pre-calculus 12

CHAPTER 7 – Exponential Functions/Equationsq

Day	Section	Homework
1	7.1/7.2 Characteristics/Transformations of Exponential Functions	
2	7.1/7.2 Applications of Exponential Functions	
3	7.3 Solving Exponential Equations	
	Chapter 7 Review	

- ***This schedule is subject to change – if you have been away check with Mr. Q to keep on top of any changes***
- ***Remember that all the above assignments (including the REVIEW) are due in a duotang on the day of the test***
- ***Please follow the HW criteria***

TEST DAY:____________________________________

PreCalc 12

Chapter 7 – Exponential Functions

Day 1 (7.1 p.334)– Characteristics of Exponential Functions

Review of the basic exponent laws

$b^x b^y =$ ________ $\qquad \frac{b^x}{b^y} =$ ________ $\qquad b^{-x} =$ ________

$\frac{1}{b^x} =$ ________ $\qquad \left(\frac{a}{b}\right)^{-x} =$ ________ $\qquad b^{\frac{m}{n}} =$ ________

$(b^x)^y =$ ________

Examples

1. $(2x^3)^4$

2. $x^{\frac{1}{2}} \cdot x^{\frac{3}{4}}$

3. 4^{-3}

4. $\frac{1}{5^{-3}}$

5. $4^{\frac{3}{2}}$

6. $27^{-\frac{2}{3}}$

7. $16^{\frac{3}{4}} \div 27^{\frac{1}{3}}$

8. $\left(\frac{81}{16}\right)^{-0.75}$

9. Express as a power of 2:

$a)$ 16^{3x-1}

b) $\frac{(4^x)(2^{x+3})}{8^{x+1}}$

Exponential Function -

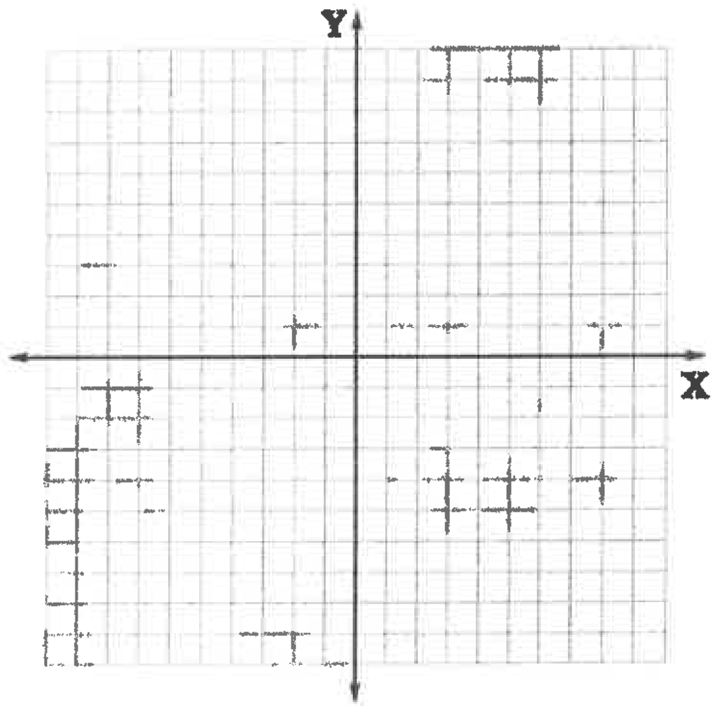
Exponential functions model various situations of growth and decay in the natural world.

We will first explore a basic exponential function of the form:

$y = c^x, c > 1$

ex. $y = 2^x$

x	y



Characteristics to be noted:

Domain:________________________

Range:________________________

Y- intercept:____________________

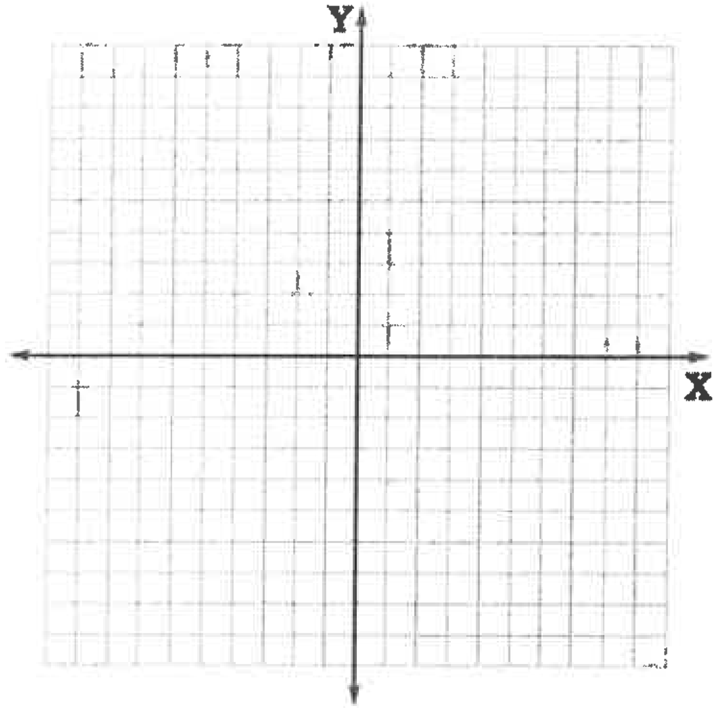
Asymptotes:____________________

Now we explore a basic exponential function with base between 0 and 1

$y = c^x,\ 0 < c < 1$

ex. $y = (\frac{1}{2})^x$

x	y



Characteristics to be noted:

Domain:________________________

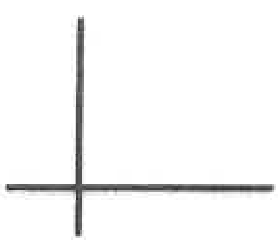
Range:_________________________

Y- intercept:____________________

Asymptotes:_____________________

In general:

$y = c^x, c > 1$



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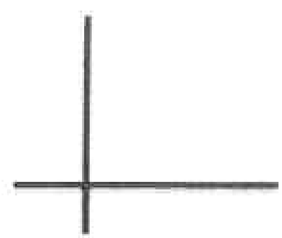
$y = c^x,\ 0 < c < 1$



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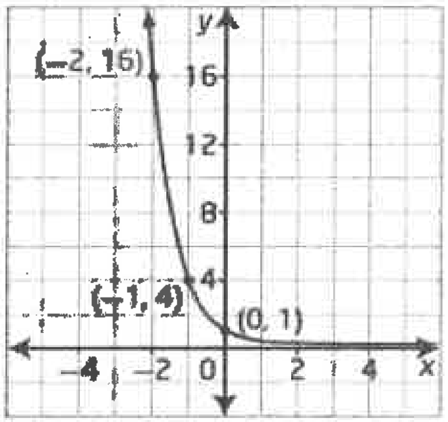
What would the graph look like if c=1 , c<0 ?

Example 1

Write the Exponential Function Given Its Graph

What function of the form $y = c^x$ can be used to describe the graph shown?



7.2 (p.346) Transformations of Exponential Functions

The graph of the function $f(x) = a(c)^{b(x-h)} + k$

is obtained by applying transformations to the base graph
$y = c^x$, *where* $c>0$ *and* $c \neq 1$

Review of the affect of the parameters: a, b, h, k

a: ______________________________

b: ______________________________

h: ______________________________

k: ______________________________

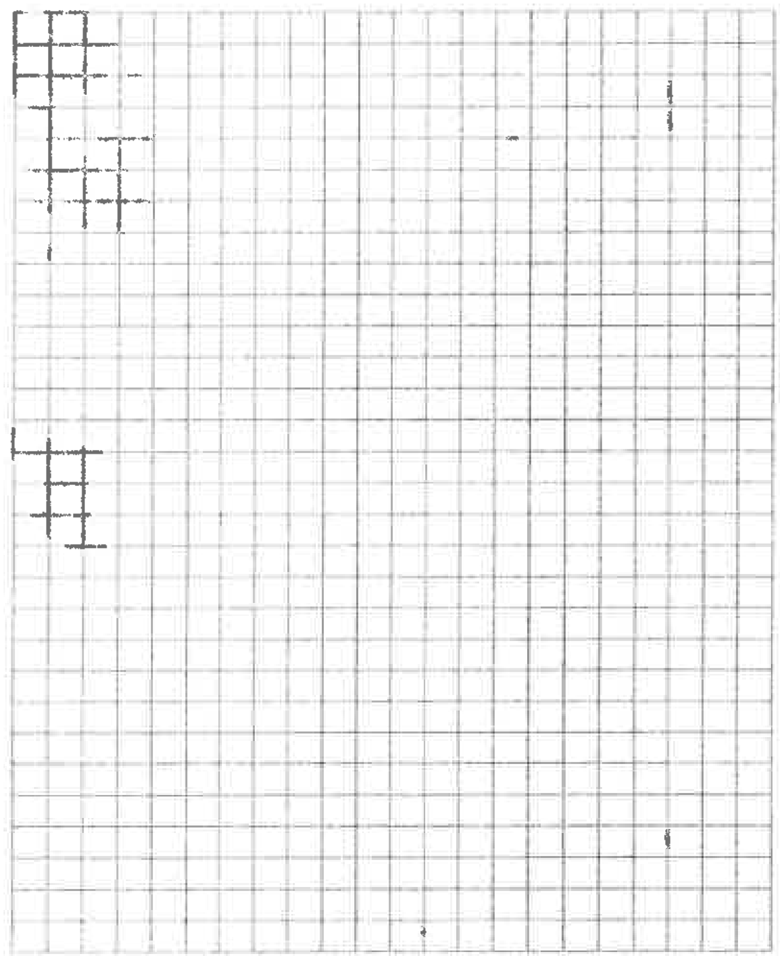
Example 2

Transform the graph of $y = 4^x$ to sketch the graph of
$$y = 4^{-2(x+5)} - 3$$

(i) State the parameters and describe corresponding transformations
(ii) Create a table to show what happens to the given points
(iii) Sketch the graphs of the base and transformed function
(iv) Describe the effects on the domain, range, equation of the horizontal asymptote and intercepts.

SOLUTION:

$y = 4^x$	$y = 4^{-2(x+5)} - 3$



Day 2 (7.1/7.2)– Applications of Exponential Functions

The following scenarios provide some examples of how exponential functions can be used to model growth and decay in the natural world.

You will first construct the function, and then use it to answer related questions. Pay attention to how the transformations relate to the information provided.

You will hopefully notice the similarity in these functions to this generalized form:

$$\boldsymbol{Final = Initial(Base)^{Exponent}}$$

Example 1

A certain culture of bacteria doubles every 20 hr. The initial count shows 300 bacteria.

a) Write an exponential function that models the given condition.

b) Approximately how many bacteria will there be in 72 hr?

c) How many bacteria were there 2 days prior to the count?

d) Graph the function using your graphing calculator.

Example 2

Saccharomyces cerevisiae is the scientific name for yeast, a single-celled fungal microorganism. A laboratory technician at Pacific Western Brewery is studying some cells that exhibit a quadrupling time of 30 hr. The initial count is 500.

a) Write an exponential function that models the given condition.

b) Approximately how many yeast cells will there be in 4 days?

c) How many cells were there 3 days ago?

d) Graph the function using your graphing calculator.

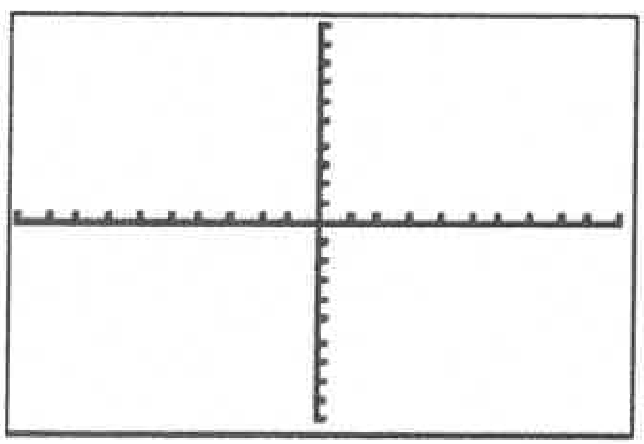
Example 3

Recall the compounding interest formula: $A = P\left(1 + \frac{i}{n}\right)^{nt}$

An investment of $1200 is earning 5.4% compounded quarterly.

a) Write an exponential function that represents the amount of the investment A(t) in dollars.

b) Graph the function using your graphing calculator and determine how many years it will take for the amount of the investment to triple.



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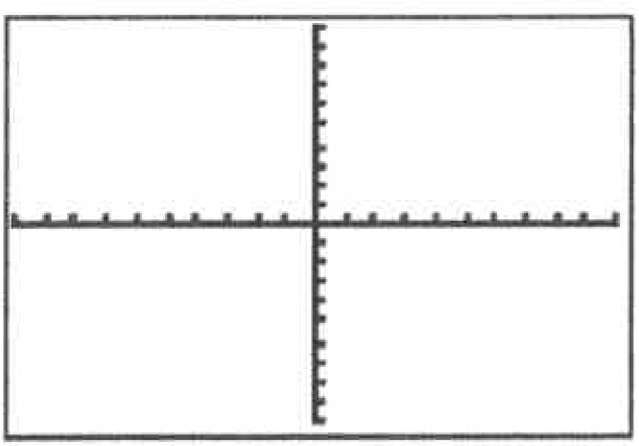
Example 4
The intensity of the light below the surface of a particular lake is reduced by 6% for every metre below the surface.

a) Write an exponential function that represents the intensity of light at any depth below the surface.

b) What percent of the original intensity of the light remains 10m below the surface?

c) Use your graphing calculator to determine how far below the surface the light has to travel for the intensity to be 30% of the surface intensity.



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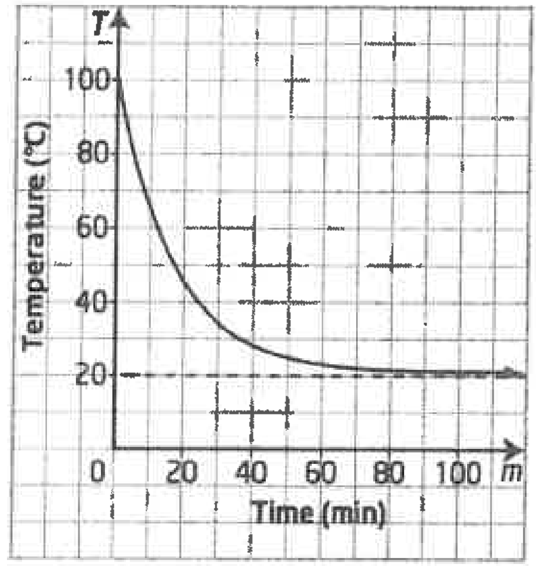
Example 5

A cup of water is heated to 100 °C and then allowed to cool in a room with an air temperature of 20 °C. The temperature, T, in degrees Celsius, is measured every minute as a function of time, m, in minutes, and these points are plotted on a coordinate grid. It is found that the temperature of the water decreases exponentially at a rate of 25% every 5 min. A smooth curve is drawn through the points, resulting in the graph shown.



a) What is the transformed exponential function in the form $y = a(c)^{b(x-h)} + k$ that can be used to represent this situation?

b) Describe how each of the parameters in the transformed function relates to the information provided.

Day 3 (7.3 p.358) - Solving Exponential Equations

In this section we explore solving exponential equations. These are equations where the ______________________________________.

Take the following equation as an example:

$$2^{4x+6} = (2^{x-2})^2$$

If the ______________________ then we can solve directly by employing the exponent laws.

If the bases are **not** the same then ______________________________

____

Example 1

$$8^{4x+1} = 16^{5x-1}$$

Example 2

$$9^{3x+1} = \left(\frac{1}{27}\right)^{4x-2}$$

Example 3

$$\frac{16^{x+3}}{8^{4x-7}} = 32^{5x+1}$$

Example 4

$$\left(\frac{4^{2x-1}}{8^{x+2}}\right)^2 = 16^{3-4x}$$

If exponential equations have different bases that **cannot** be rewritten to the same base then some options for solving include:

- **Trial and error ☹**
- **Graphing calculator technology**
- **Logarithms (next unit)☺**

Now follow some more applications similar to previous section. However, after building the function, we will try and answer the questions by solving algebraically.

Example 5

To determine whether a person has a thyroid gland deficiency, radioactive iodine with a ***half-life*** of 8.2 days is injected into the bloodstream. A healthy thyroid gland absorbs the radioactivity.

a) Write an exponential function that models the amount of radioactive iodine that should be present in the thyroid gland of a healthy person after any number of days.

b) After how long should only 25% of the radioactive iodine be present in the thyroid gland of a healthy person?

Example 6

A bacterial culture starts with 6000 bacteria. After 3 hr the estimated count is 162 000. What is the ***tripling*** time of this culture?

Example 7

Once inside the human body an infectious virus doubles every 6 min. The first symptoms of infection occur when there are about 4^{30} of the viral particles present in the body. If a person is infected with a single virus now, how long will it take for the first symptoms to appear?